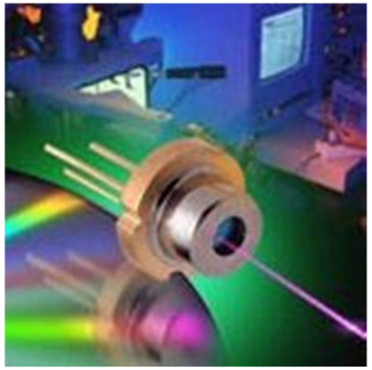
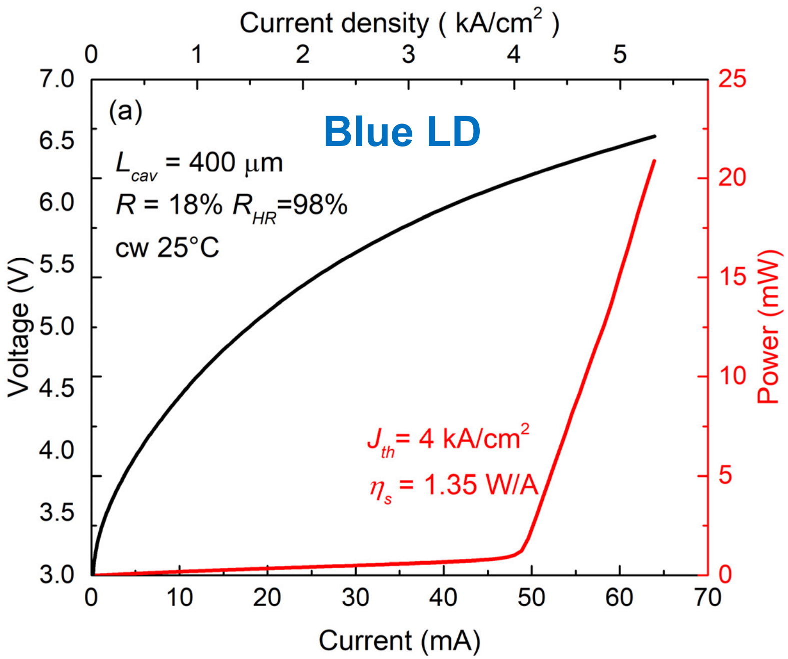
Lecture 12 – 14/05/2025

Laser diodes

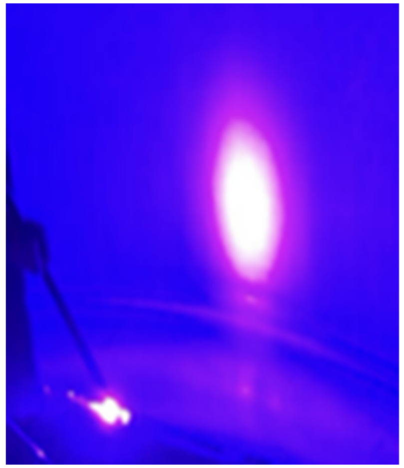
- Generalities
- Electrical injection
- Material gain: bulk and QW cases
- Laser oscillations
- Output power



Laser diodes



Far-field pattern



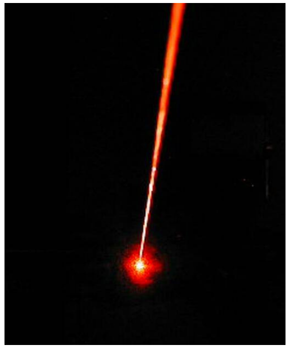
Laser: a clear threshold is observed in the *L-I* curve (+ far-field pattern)

⇒ Light amplification

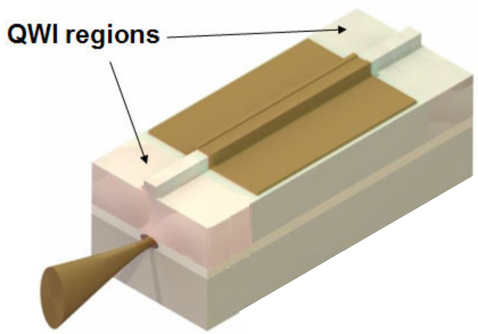
Light Amplification** by **Stimulated Emission of Radiation

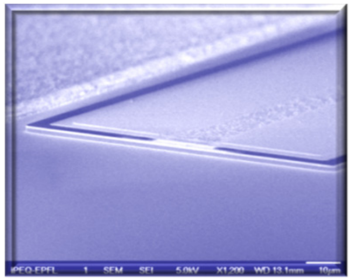
Semiconductors: a brief overview

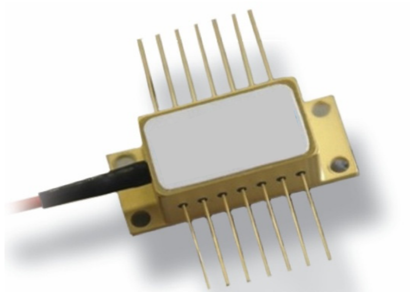
1980's → **GaAs based optoelectronics**



1970
1st laser diode
λ ~780 nm
J_{thr}~ 4.3 kA/cm^2
Ioffe, Russia







2000

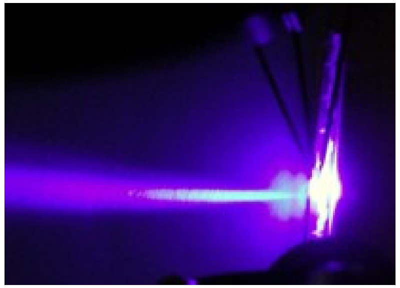
CD, DVD, Telecom

BUT light emission limited to the **Red** and **IR**

Semiconductors: a brief overview

1990's → GaN short-wavelength optoelectronics

1993



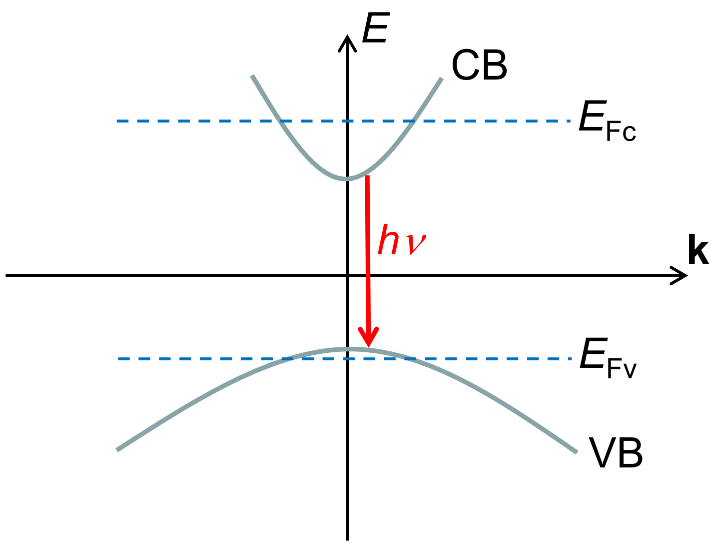
UV, blue, and green LEDs
High density DVD, color displays

2003

Demonstration of reliable (i.e., long lifetime) green semiconductor laser diodes with a high production yield on the way (available from Nichia. Inc.)!

Electrical injection

- Both the valence and the conduction bands get more and more filled upon increasing current injection
- The carrier populations are described by the quasi-Fermi levels E_{Fc} and E_{Fv}



$$f_c(E) = \frac{1}{\exp\left(\frac{E - E_{F_c}}{k_B T}\right) + 1}$$

$$f_v(E) = \frac{1}{\exp\left(\frac{E - E_{F_v}}{k_B T}\right) + 1}$$

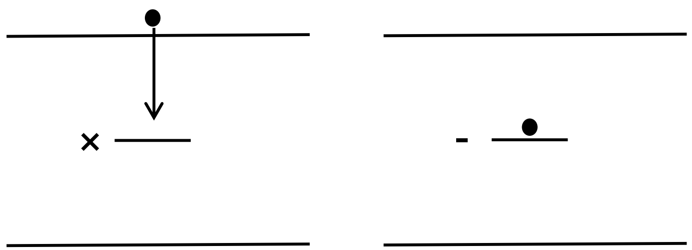
Note that here $f_v(E)$ describes the evolution of the electron population in the valence band!

Electrical injection

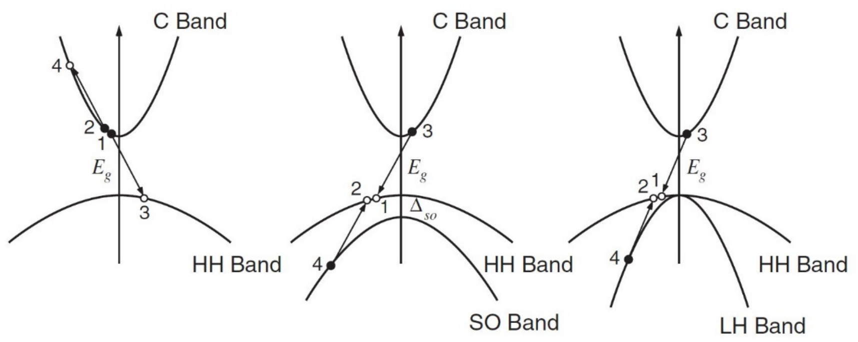
Different paths for electron-hole recombinations

- Non-radiative $A_{nr}n$
- Spontaneous Bn^2 — B bimol. coeff. ~10^{-12}-10^{-10} cm^3s^{-1}
- Auger Cn^3
- Stimulated $R_{st}s$ — with $R_{st} = - R_{abs}$ and s the density of stimulated photons

Stimulated recombination rate



Shockley-Read-Hall recombinations



Auger recombinations

Electrical injection

Below threshold – spontaneous emission regime (s = 0)

$R_{tot}\,V(= Sd) = J/q\; S \Rightarrow R_{tot} = J/(qd) = A_{nr}n + Bn^2 + Cn^3$

d = active region thickness

$1/\tau_{nr} = A_{nr} + Cn^2$ Non radiative recombinations

$1/\tau_r = Bn$ Radiative recombinations

with $1/\tau_{tot} = 1/\tau_r + 1/\tau_{nr}$

Finally $R_{tot} = (1/\tau_{nr} + 1/\tau_r)n = J/(qd)$ and $n = J\tau_{tot}/(qd)$

One recalls that the internal quantum efficiency (IQE) is given by

$$\eta_i = \frac{\tau_{tot}}{\tau_r} = \frac{\tau_{nr}}{\tau_{nr} + \tau_r} = \frac{Bn}{A_{nr} + Bn + Cn^2}$$

and the photon flux is given by $\phi = \eta_{inj}\eta_i\, J/q$

Often taken equal to 1 (assumption we make from now on)!

Electrical injection

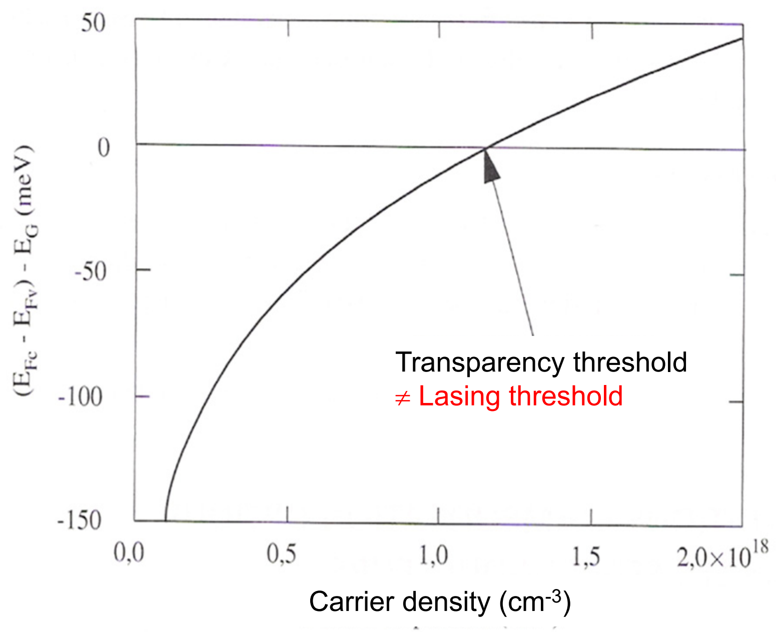
$$n = J\tau_{tot}/qd$$

A few remarks:

- *The effective recombination lifetime τ_{tot} depends on the carrier density*
- *The Auger recombination term is significant only at high injection*
- *The carrier density depends on the active region thickness*

- homojunction: $d = L_{Dn} + L_{Dp}$ (1-10 μm)
- heterojunction: d = 100 nm
- quantum well: d = 1-10 nm

Stimulated emission



The material becomes transparent when

$$E_{F_c} - E_{F_v} = E_g$$

True for a bulk SC!

Minimum requirement to fulfill the Bernard-Duraffourg condition (cf. fall semester, **Lecture 12**)

Transparency threshold in GaAs

d = 1 μm ⇒ J_{tr} ~16 kA/cm² bulk (1960)

d = 100 nm ⇒ J_{tr} ~1.6 kA/cm² heterojunction (1970) (≡ double heterostructure (DHS))

d = 10 nm ⇒ J_{tr} ~160 A/cm² quantum well (1980) (≡ separate confinement heterostructure (SCH))

Stimulated emission

n increases with the current

$$n=\int_{E_c}^{\infty}\rho_c(E)\frac{1}{\exp\left(\frac{E-E_{F_c}}{k_B T}\right)+1}dE \qquad E_{Fc}\,(E_{Fv})\uparrow$$

and the absorption is given by

$$\alpha(\omega)=-\gamma(\omega)=\alpha_0(\omega)\left[f_v(\hbar\omega)-f_c(\hbar\omega)\right]$$ where γ is the gain

When α becomes negative, stimulated emission becomes possible (but one extra condition must be fulfilled for achieving lasing)

$$f_c(\hbar\omega)\geq f_v(\hbar\omega) \quad \Rightarrow \quad \boxed{E_{Fc}-E_{Fv}\geq\hbar\omega\geq E_g}$$

Bernard-Duraffourg condition

Light gets amplified only once the Bernard-Duraffourg condition is fulfilled, i.e., when the semiconducting medium exhibits optical gain!

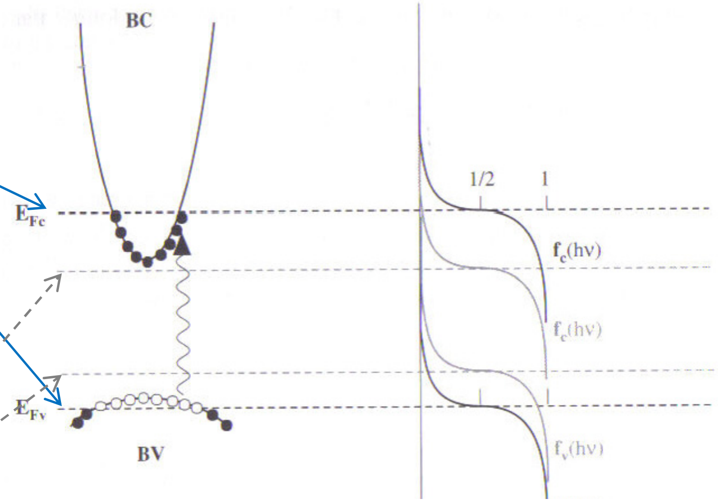
⇒ Necessary condition for the achievement of lasing in a semiconducting medium (⚠ but it is not a sufficient one)

Stimulated emission

$$E_{Fc} - E_{Fv} > \hbar\omega > E_g$$

- Strong excitation
  - ✓ At least one of the bands is degenerate
  - ✓ All the states satisfying the *B-D* inequality are "fully occupied", i.e. the SC is transparent for those λ!
- Weak or moderate excitation
  - ✓ None of the bands are degenerate, i.e. $n < N_C$ and $p < N_V \Rightarrow$ use of Boltzmann approximation
  - ✓ $\Rightarrow$ photon absorption is still at play since there are available states in the CB where e^- from the VB can be promoted

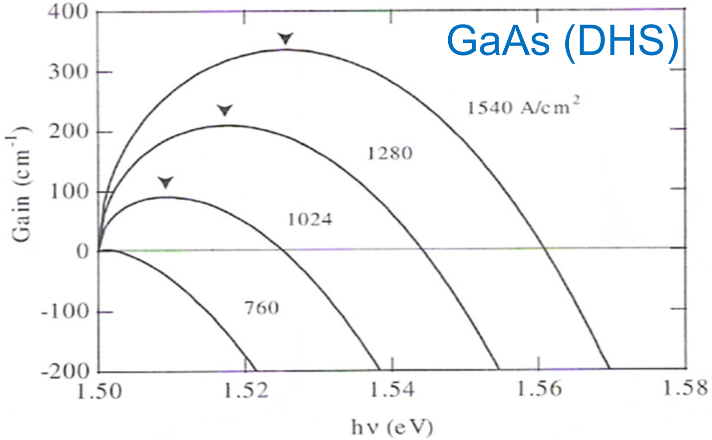
System driven out of equilibrium



Cf. slide 17 of Lecture 9 and related comments!

Material gain (bulk case)

Gain curves

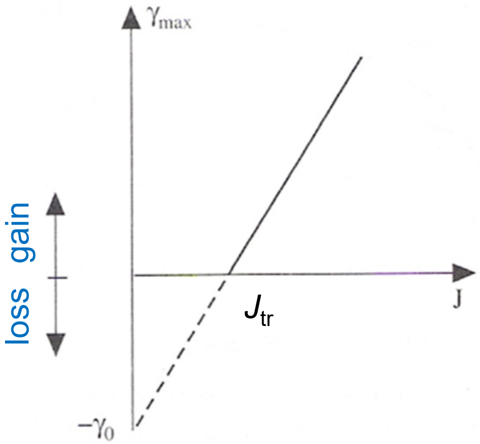


In a semiconductor, the material gain can also be expressed as

JDOS

$$\gamma(h\nu) = \frac{\lambda^2}{8\pi\tau_r}\rho_j(h\nu)\left[f_c(h\nu) - f_v(h\nu)\right]$$

Increase and broadening of the gain region with the current



In a bulk system, the maximum gain varies linearly with the carrier density n above the transparency carrier density n_{tr}

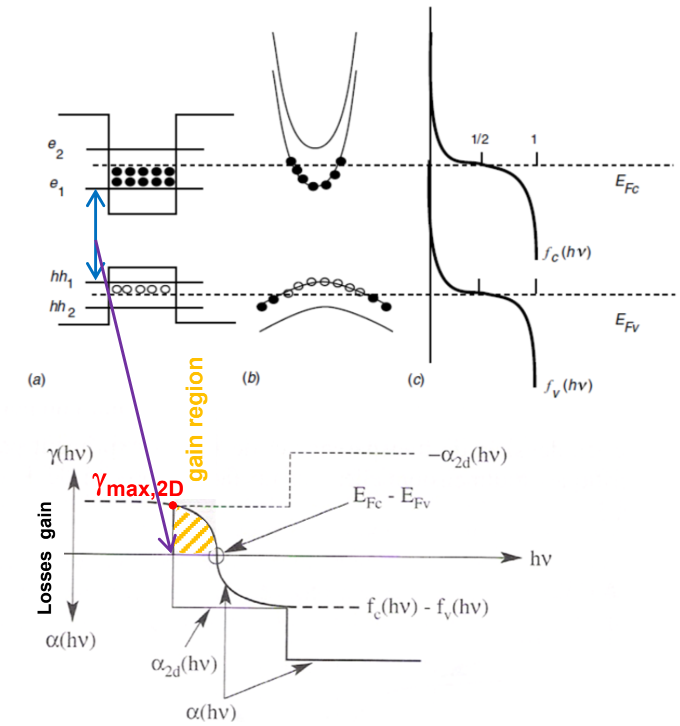
$$\gamma_{max} = \gamma_0\left(\frac{n}{n_{tr}} - 1\right)$$

$$\Rightarrow \gamma_{max} \approx \gamma_0\left(\frac{J}{J_{tr}} - 1\right)$$

knowing that $J_{tr} = \frac{qd}{\tau_{tot@tr}} n_{tr} = \frac{qd}{\eta_i \tau_{r@tr}} n_{tr}$

⚠ Keep in mind the nonlinear relationship between the current density and the carrier density (⇒ ABC-model)!

Material gain (QW case)



Cf. Lecture 12, fall semester for 3D case!
+
Rosencher-Vinter's book

Case of quantum wells:

Quasi-Fermi levels derived from

$$n = \int_{e_1' = E_g + e_1}^{\infty} \rho_{2D}^{e}(E) f_c^1(E) dE$$

$$p = \int_{-\infty}^{-hh_1} \rho_{2D}^{hh}(E)\left(1 - f_v^1(E)\right) dE$$

Note: only the ground states are populated at the transparency threshold

Material gain for the e_1-hh_1 transition

$$\gamma(h\nu) = \alpha_{2D}\left[f_c^1(h\nu) - f_v^1(h\nu)\right]\theta\left(h\nu - E_g - e_1 - hh_1\right)$$

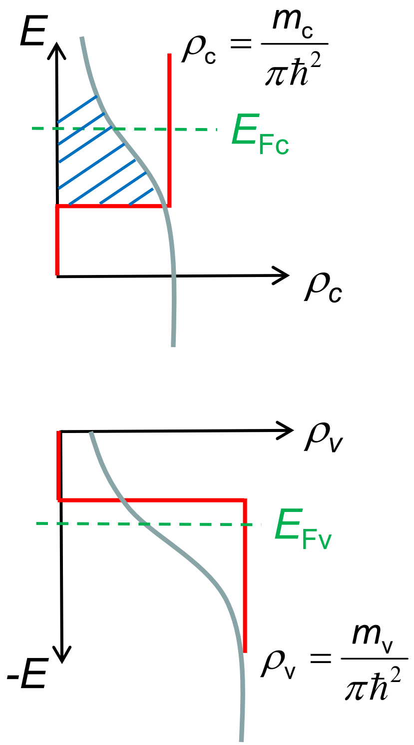
with $\alpha_{2D}(\hbar\omega) = \frac{\pi q^2 x_{vc}^2 \omega}{\varepsilon_0 n_{op} c d}\rho_{2D}(\hbar\omega) = \frac{2\pi q^2 x_{vc}^2 m_r^*}{\varepsilon_0 \hbar^2 n_{op} \lambda d}$

2D-JDOS

Note: The zero of energy is taken at the top of the valence band

Material gain (QW case)

Quantum well:



$$n = \int_{e_1' = E_g + e_1}^{+\infty} \rho_c \frac{1}{\exp\left(\frac{E - E_{F_c}}{k_B T}\right) + 1} dE \Rightarrow n = n_c \ln\left(1 + \exp\left(\frac{E_{F_c} - e_1'}{k_B T}\right)\right)$$

$$n_c = \rho_c k_B T$$

$$p = \int_{-\infty}^{-hh_1} \rho_v \frac{1}{\exp\left(\frac{E_{F_v} - E}{k_B T}\right) + 1} dE \Rightarrow p = p_c \ln\left(1 + \exp\left(-\frac{hh_1 + E_{F_v}}{k_B T}\right)\right)$$

$$p_c = \rho_v k_B T \quad \text{and} \quad n_c / p_c = \rho_c / \rho_v = m_c / m_v = 1/R$$

n_c and p_c are the 2D critical densities

⚠ not to be confused with the reflectivity coefficient!

Note: The zero of energy is taken at the top of the valence band!

Material gain (QW case)

Quantum well:

The material gain exhibits its maximum at the absorption edge due to the staircase profile of the 2D-DOS

Maximum gain for $\hbar\omega = \Delta E = e_1^{'} + hh_1 \Rightarrow E_{\text{c}}(\hbar\omega) = e_1^{'}$ and $E_{\text{v}}(\hbar\omega) = -hh_1$

$$\Rightarrow \quad \gamma_{\max} = \gamma_0\left(f_{\text{c}}(\Delta E) - f_{\text{v}}(\Delta E)\right) = \gamma_0\left(\frac{1}{\exp\left(\frac{e_1^{'} - E_{\text{F}_\text{c}}}{k_{\text{B}}T}\right)+1} - \frac{1}{\exp\left(-\frac{hh_1 + E_{\text{F}_\text{v}}}{k_{\text{B}}T}\right)+1}\right) \text{ with } \gamma_0 = \alpha_{\text{2D}}$$

$$E_{\text{F}_\text{c}} - e_1^{'} = k_{\text{B}}T\ln\left(e^{n/n_\text{c}} - 1\right)$$

$$-\left(hh_1 + E_{\text{F}_\text{v}}\right) = k_{\text{B}}T\ln\left(e^{p/p_\text{c}} - 1\right)$$

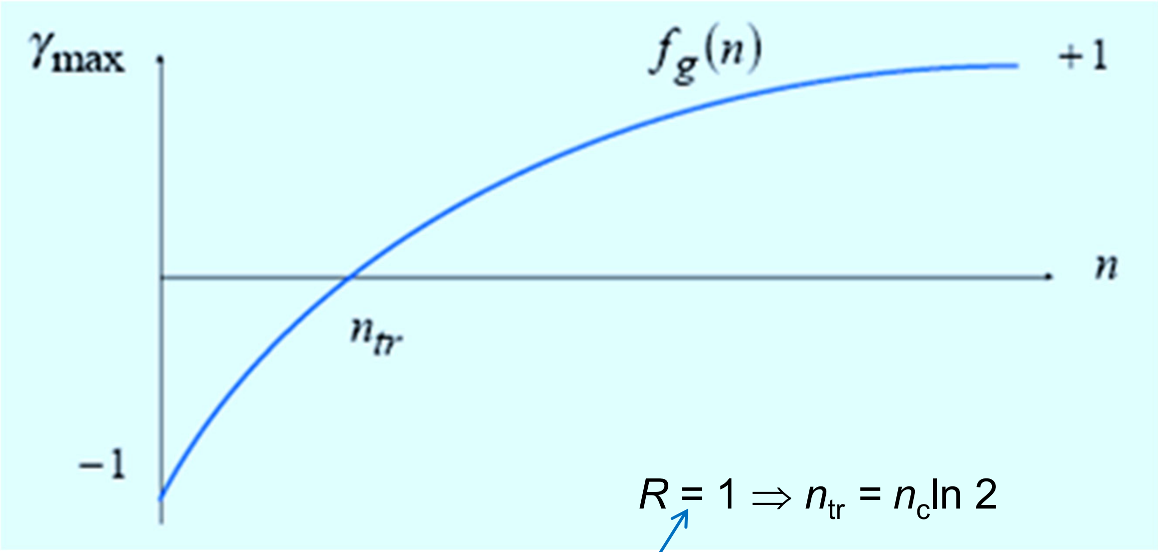
$\gamma_{\max} = \gamma_0 f_{\text{g}}(n)$ with $f_{\text{g}}(n) = 1 - e^{-n/n_\text{c}} - e^{-n/p_\text{c}}$ and $n = p$ due to electrical neutrality

$f_{\text{g}}(n) = 1 - e^{-n/n_\text{c}} - e^{-n/Rn_\text{c}}$ with $R = \dfrac{m_\text{v}}{m_\text{c}}$

Material gain (QW case)

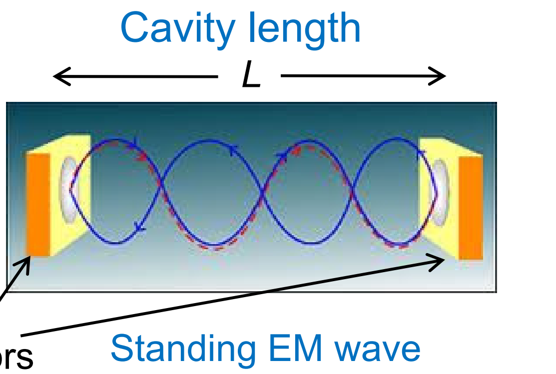
Quantum well:

Transparency when $f_g(n) = 1 - e^{-n/n_c} - e^{-n/Rn_c} > 0$



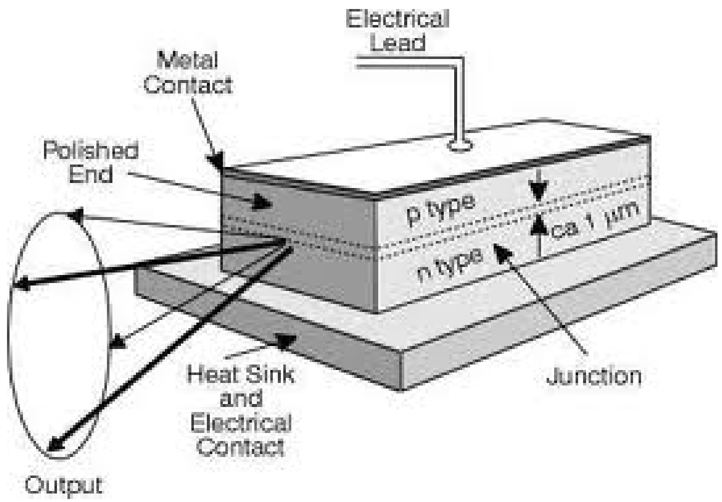
Fictitious but illustrative example!

Edge-emitting laser diode



≡ cleaved facets (along atomic planes)

Resonant cavity ⇒ optical feedback



$$R_m = (n_{sc}-1)^2 / (n_{sc}+1)^2$$

E.g., for GaAs, $R_m = 0.32$

Laser oscillations

Condition for lasing:

$$\text{Modal gain} = \text{losses} \Rightarrow \Gamma\gamma_{thr}(h\nu) = \alpha_p + 1/(2L) \times \ln(1/R_1R_2)$$

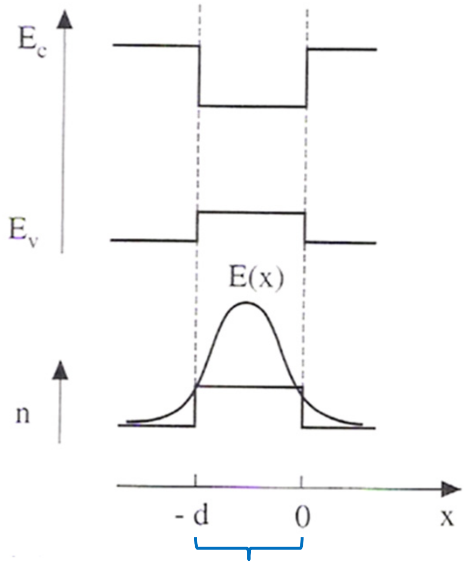
Mirror losses

Modal gain

= confinement factor × material gain

Parasitic or intrinsic losses

Optical waveguide:



Extent of the gain medium

Overlap between optical mode and active region (gain medium, e.g., QWs)

The confinement factor (Γ) is given by:

$$\Gamma = \frac{\int_{-d}^{0} |E(x)|^2 dx}{\int_{-\infty}^{+\infty} |E(x)|^2 dx}$$

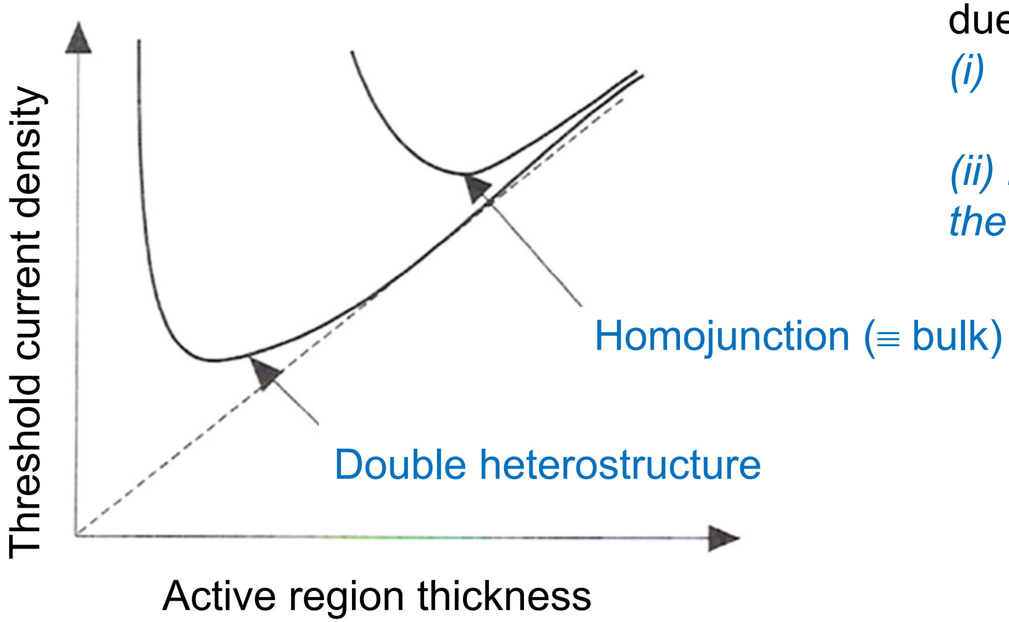
For a QW-based LD (SCH), Γ = 0.5-5%

For a double heterostructure (DHS), Γ ~ 1 ($d \approx$ 100 nm)

Laser oscillations

Condition for lasing:

$$\Gamma\gamma_{thr}(h\nu) = \alpha_p + 1/(2L) \times \ln(1/R_1R_2)$$



Lower J_{thr} of DHS-LDs vs. homojunction ones due to:

(i) better confinement of carriers

(ii) better overlap between the gain region and the EM field

Output power

1. Below the transparency threshold: weak contribution due to spontaneous emission emitted through the outcoupling mirror/facet

2. Above the transparency threshold: amplified spontaneous emission (ASE), $\Rightarrow$ *superluminescence*

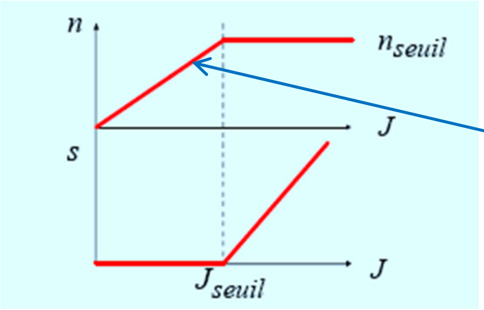
$$n_{tr} \leq n \leq n_{thr}$$

3. When $\Gamma\gamma$ (modal gain) equals the losses: laser oscillations start

Important: once the lasing threshold is reached, the carrier density is clamped (constant)

each newly added electron gives rise to 1 stimulated photon (x IQE)

The feedback effect causes the carrier density to clamp in order to keep the gain at its threshold value!



Approximate description since *n* is not a linear function of *J*!

Output power

Equation

$$J/(qd) = A_{nr}n + Bn^2 + Cn^3 + R_{st}s$$

R_{st}: stimulated emission rate
s: density of stimulated photons

$n = n_{thr}$ above threshold:

$$J/(qd) = n_{thr}/\tau_{thr} + R_{st}s$$

$$J_{thr} = q\, d\, n_{thr}/\tau_{thr} \ (s = 0)$$

$$\Rightarrow R_{st}s = (J - J_{thr})/qd$$

The stimulated emission rate is given by $R_{st} = \gamma_{thr}\, c/n_{op} = 1/(\Gamma\tau_{cav})$ (cf. rate eqs. in Lectures 13 & 14)

Finally, the density of photons inside the cavity is given by:
$s = (n_{op}/(qd\gamma_{thr}c))(J-J_{thr})$,

$\Rightarrow$ all the current above threshold is converted in stimulated emission!
In practice, one must also account for nonradiative channels (i.e., the IQE is included)

$$s = \eta_i(n_{op}/(qd\gamma_{thr}c))(J-J_{thr})$$

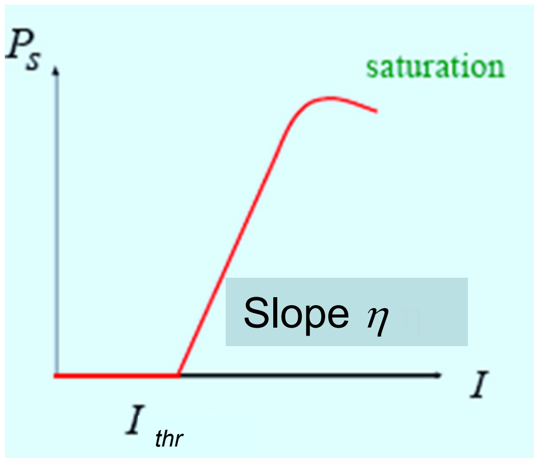
Output power

$\tau_{cav} = n_{op}/(\Gamma \gamma_{thr} c) \Rightarrow s = \eta_i (\Gamma \tau_{cav}/qd)(J-J_{thr})$ $\gamma_{thr} = (\alpha_m + \alpha_p)/\Gamma$ $c' = c/n_{op}$

$$P_s = \underbrace{\begin{pmatrix}\text{Photon}\\ \text{energy}\end{pmatrix}}_{h\nu} \underbrace{\begin{pmatrix}\text{Photon}\\ \text{density}\end{pmatrix}}_{\eta_i \frac{\Gamma \tau_{cav}}{qd}(J-J_{thr})} \underbrace{\begin{pmatrix}\text{Effective}\\ \text{mode volume}\end{pmatrix}}_{Sd/\Gamma} \underbrace{\begin{pmatrix}\text{Photon}\\ \text{escape rate}\end{pmatrix}}_{c'\alpha_m}, \quad I = JS$$

$$P_s = \eta_i \frac{\alpha_m}{\alpha_p + \alpha_m} \frac{h\nu}{q}(I - I_{thr}) \Rightarrow P_s = \eta \frac{h\nu}{q}(I - I_{thr})$$

where we assumed that the injection efficiency = unity



Wall plug efficiency (WPE)

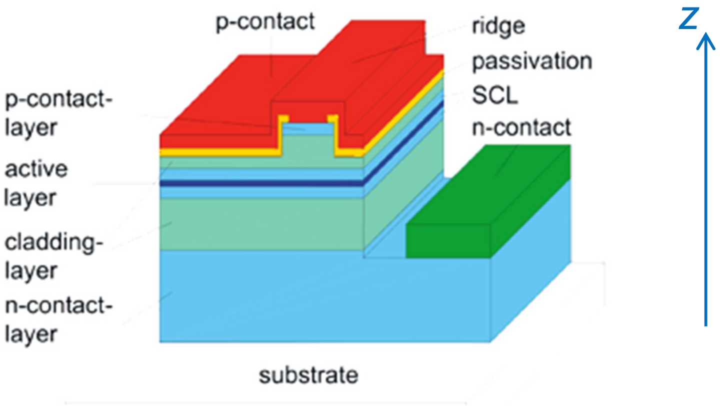
$P_{el} = V \times I \approx h\nu/q \times I$ (assuming cold carrier injection)

$P_s/P_{el} = \text{WPE} \approx \eta(1 - I_{thr}/I)$

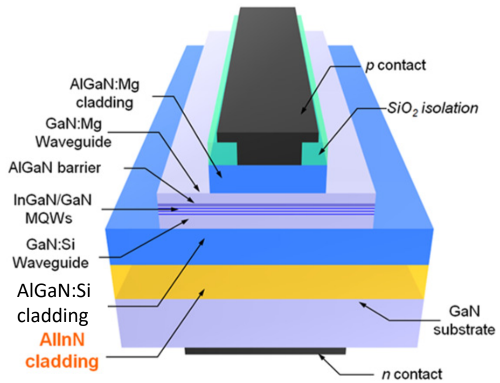
External quantum efficiency

\> 30% for semiconductor lasers
1% for gas lasers

Edge-emitting laser diode

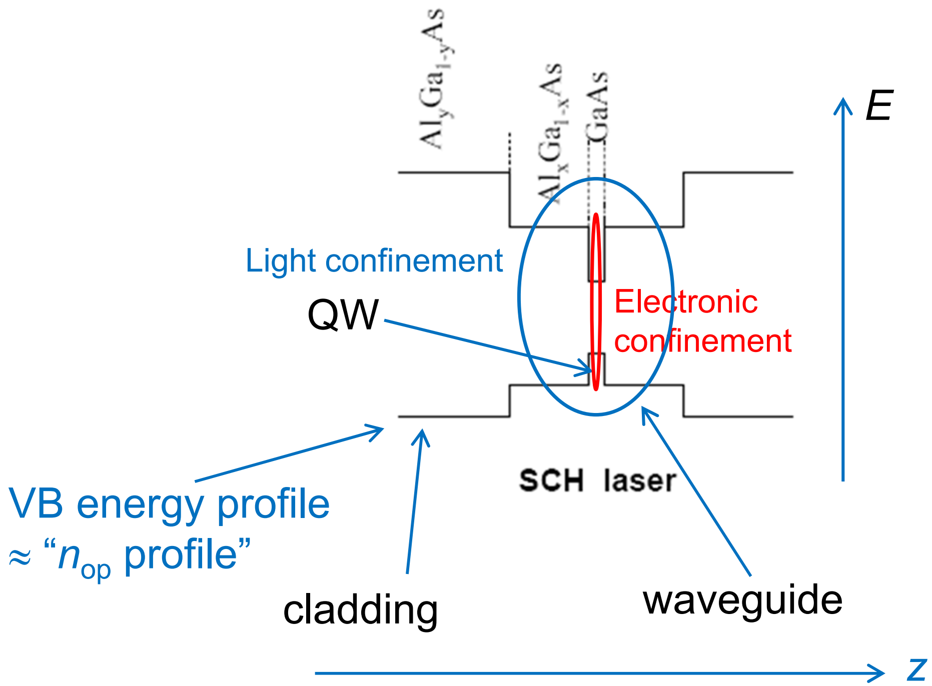


0.2-1 μm x 2-5 μm x 200-1000 μm



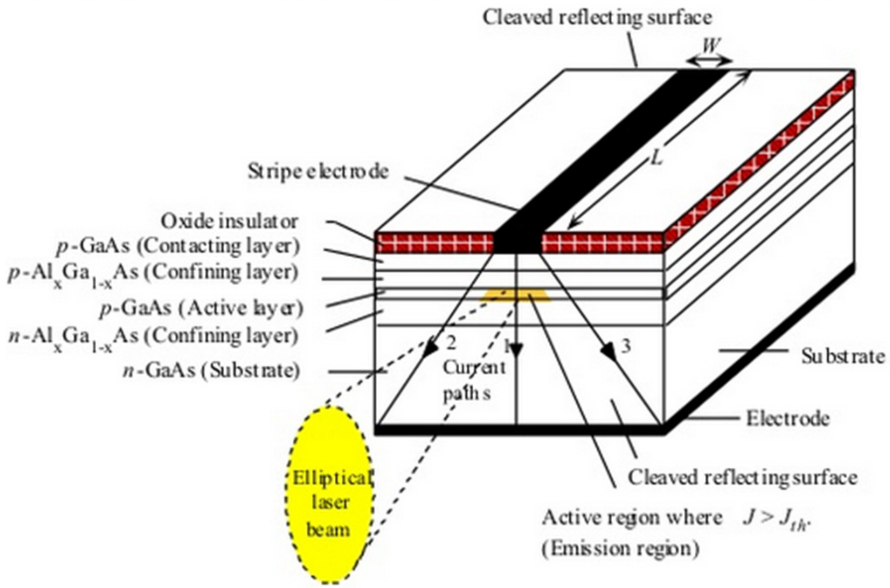
Index-guided LD structures (600 x 2 μm^2)

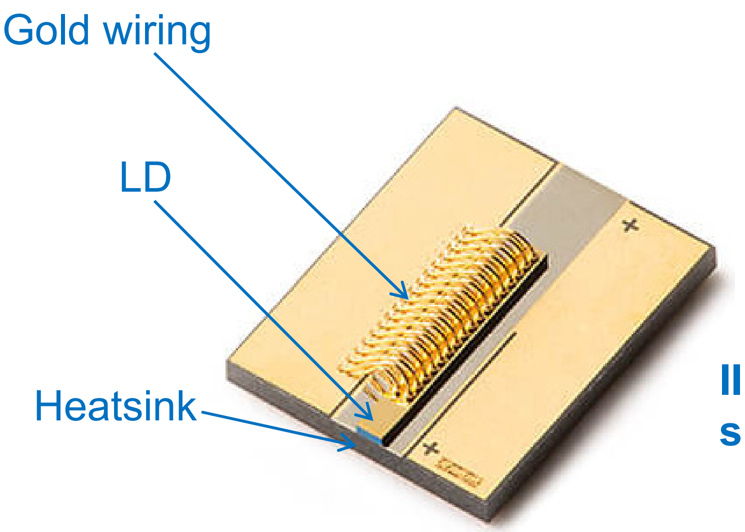
"LED" + cavity (mirrors)



SCH: separate confinement heterostructure

Edge-emitting laser diode





II-VI Inc. high output power (12 W, cw) near-infrared single mode edge emitting laser diode (single QW)

Edge-emitting laser diode

Internal efficiency

The output power is given by:

$P_s = \eta(h\nu/q)(I-I_{thr})$ with $\eta = \eta_i\alpha_m/(\alpha_m+\alpha_p) = \eta_i \ln(1/R_m)/[\alpha_p L+\ln(1/R_m)]$

with R_m the mirror reflectivity (taken identical for both sides)

External differential quantum efficiency:

$$\eta_d = \left(\frac{dP_s}{dI}\right)\left(\frac{q}{h\nu}\right) \quad (I > I_{thr})$$

Thus $$\eta_d = \frac{\eta_i}{\left[1+\alpha_p L \Big/ \ln\left(\frac{1}{R_m}\right)\right]}$$

Internal quantum efficiency